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TRANSFER FUNCTION OF A FEEDBACK AMPLIFIER CONSISTING OF "N" IDENTICAL CASCADES

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The transfer function of a feedback amplifier of n identical cascades has a simple analytical expression of closed form if the transfer function of each cascade has the form:

$$Q(t) = Q(1 - e^{-t/T}), \tag{1}$$

where Q is the static coefficient of transfer (see A. V. Mikhaylov, ZhTF, 9, 1, 1939). By transfer functions of a certain transmitting or transfer system we mean the dependence of output signal upon time, the signal being due to a unit-step excitation at the input of the amplifier.

$$U_i(t) = \begin{cases} 1 \text{ for } t > 0, \\ 0 \text{ for } t < 0 \end{cases}$$

The dynamic coefficient of transfer of one cascade represented as a Laplace-Carson transfer function is

$$Q(p) = \frac{Q}{1 + Tp} \tag{2}$$

The dynamic coefficient of transfer of an amplifier of n cascades without feedback is

$$Q_n(p) = Q^n(p) = \frac{Q^n}{(1 + Tp)^n} \tag{3}$$

Upon introducing into an amplifier a feedback circuit which feeds into the input of the amplifier B-th part of the signal from the output, we obtain the following coefficient of transfer of an amplifier

$$Q_3(p) = \frac{Q^n(p)}{1 - \beta Q^n(p)}. \tag{4}$$

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For a transfer function of an amplifier of n cascades with negative feedback ($B<0$), it is possible to obtain by a well known method the following expression, originating from (4) above:

$$Q_3(t)=\frac{Q^n}{1+\beta Q^n}+\frac{Q^n}{n}\sum_{k=1}^{n}\frac{e^{p_k t}}{Tp_k(1+Tp_k)^{n-1}}. \qquad (5)$$

Here pk are the roots of the equation

$$(1+Tp)^n+\beta Q^n=0 \qquad (6)$$

and

$$(1+Tp)=\sqrt[n]{-\beta}\,Q=Q\sqrt[n]{\beta}\left(\cos\frac{(1+2s)\pi}{n}+j\sin\frac{(1+2s)\pi}{n}\right), \qquad (7)$$

where s=0, 1,..., n-1 and s=k-1.

For n even, all the roots of equation (6) are complex paired conjugates; for n odd, one root $\left(s=\frac{n-1}{2}\right)$ is real and negative, with the others complex paired conjugates. The general expression for the roots is:

$$p_k=\frac{1}{T}\left[-1+Q\sqrt[n]{\beta}\cos\frac{(1+2s)\pi}{n}+jQ\sqrt[n]{\beta}\sin\frac{(1+2s)\pi}{n}\right]. \qquad (8)$$

After substituting in (5) these roots (8) and after pairing the terms with conjugate roots, we obtain the expression for (5), the transfer function, in the following form:

Where n is even:

$$Q_3(t)=\frac{Q^n}{1+\beta Q^n}-2\frac{Q\sqrt[n]{\beta}}{\beta n}\sum_{k=1}^{\frac{1}{2}n}\frac{\sin(\nu_k t+\varphi_k)}{\sqrt{1+\left(Q\sqrt[n]{\beta}\right)^2-2Q\sqrt[n]{\beta}\cos\frac{(1+2s)\pi}{n}}}e^{-\gamma_k t}; \qquad (9)$$

When n is odd:

$$Q_3(t)=\frac{Q^n}{1+\beta Q^n}-\frac{1}{\beta n}\frac{Q\sqrt[n]{\beta}}{1+Q\sqrt[n]{\beta}}e^{-\left(1+Q\sqrt[n]{\beta}\right)\frac{t}{T}}-$$

$$-2\frac{Q\sqrt[n]{\beta}}{\beta n}\sum_{k=1}^{\frac{1}{2}(n-1)}\frac{\sin(\nu_k t+\varphi_k)}{\sqrt{1+\left(Q\sqrt[n]{\beta}\right)^2-2Q\sqrt[n]{\beta}\cos\frac{(1+2s)\pi}{n}}}e^{-\gamma_k t}. \qquad (10)$$

Here the coefficient of damping is

$$\gamma_k=-\frac{1}{T}\left[1-Q\sqrt[n]{\beta}\cos\frac{(1+2s)\pi}{n}\right]; \qquad (11)$$

the frequ[illegible] of natural (free) oscillation is:

$$\nu_k=\frac{1}{T}Q\sqrt[n]{\beta}\sin\frac{(1+2s)\pi}{n}; \qquad (12)$$

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and the phase angle is

$$\varphi_k = \operatorname{arc\,tg} \frac{Q\sqrt[n]{\beta} - \cos\frac{(1+2s)\pi}{n}}{\sin\frac{(1+2s)\pi}{n}}. \tag{13}$$

The stability of an amplifier is determined by that condition for which the minimum coefficient of damping ηk, which holds for s equal to 0, is positive and is expressed by the following inequality:

$$Q\sqrt[n]{\beta}\cos\frac{\pi}{n} < 1, \tag{14}$$

for then the stability does not depend upon the time constant T of the cascades.

For the case of positive feedback ($\beta > 0$), the roots on the complex plane are displaced clockwise, with respect to the circle giving the roots' position for the case of negative feedback, by an angle equal to π.

The transfer function of the coefficient of damping, the frequencies of free oscillation, and their phase are expressed by formulas similar to those above, but with the substitution in them of the argument

$$\frac{(1+2s)\pi}{n} \text{ by } \frac{2s\pi}{n}.$$

The stability condition holds for $s=0$ when the root has only a real part and satisfies the inequality:

$$Q\sqrt[n]{\beta} < 1, \tag{15}$$

That is, for positive feedback and $\beta = 1$, the static coefficient of transfer Q of one cascade of stable amplifier cannot exceed unity, regardless of the magnitude of the time constant T. (See K. A. Krug's "Transitional Processes in Linear Electrical Circuits", 1948.)

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